Quantum Simulation of Gauge Theory

Scott Lawrence

with Henry Lamm and Yukari Yamauchi

(NuQS Collaboration)

Based on 1806.06649 and 190x.xxxxx

26 February 2019

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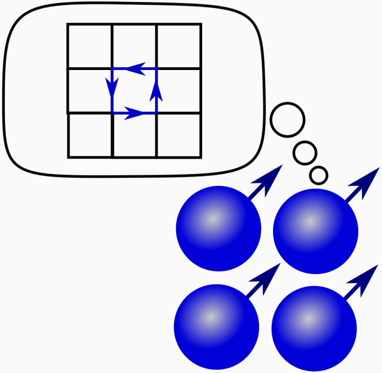
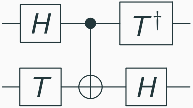
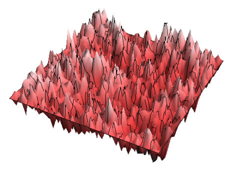
WASHINGTON, DC



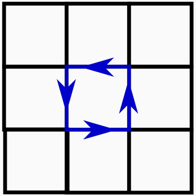
Overview

Overview of quantum algorithms





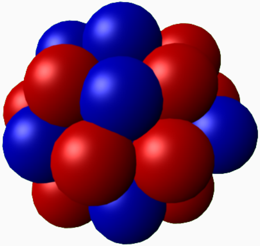
Simulations of field theories

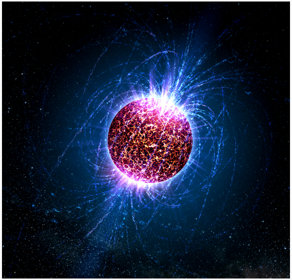


Simulating D_4 gauge theory

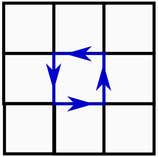
Quantum Chromodynamics

$$L = \bar{\psi}\left(i\not{D}_{\mu} - M\right)\psi - \frac{1}{4}G^{a}_{\mu\nu}G^{\mu\nu}_{a}$$



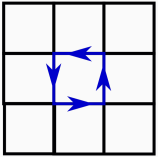


Lattice QCD



$$Z = \int_{SU(3)} \mathrm{d}U \, e^{-\int L}$$

Lattice QCD



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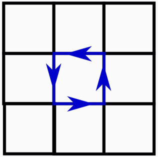
But we can't do...

- Scattering (real-time evolution via e^{-iHt})
- Finite fermion density (sign problem)
- Viscosity (real-time evolution)

Lattice QCD



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- Scattering (real-time evolution via e^{-iHt})
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Quantum computers promise all this!

From Bit to Qubit

$|0\rangle\,, \ |1\rangle \ldots$

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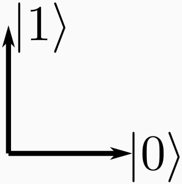
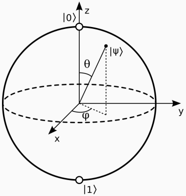
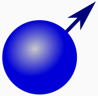
$$|0\rangle\,,\ |1\rangle \ldots\ \frac{i}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle$$

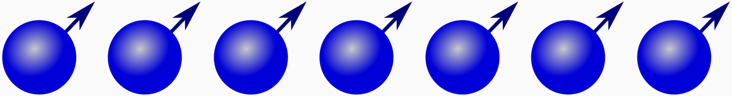




This is a spin from QM.



A Quantum Computer



Physically, there's a Hilbert space:

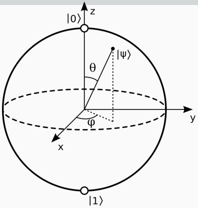
$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_1 \otimes \cdots$$

When we measure, we collapse into one of the 2^N states in the "fiducial basis".

$$|\Psi\rangle \rightarrow |0101010\rangle$$

Gates

Arbitrary one-qubit gates are ‘easy’ – can be constructed from Hadamard and $\frac{\pi}{8}$-gate.



$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \; T = \begin{pmatrix} e^{\pi/8} & 0 \\ 0 & e^{-\pi/8} \end{pmatrix}$$

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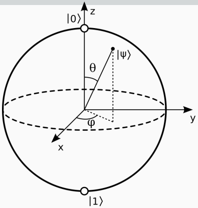
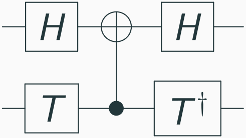


Controlled-not (CNOT) is a 2-qubit gate.



$$|00\rangle \mapsto |00\rangle$$

$$|01\rangle \mapsto |01\rangle$$

$$|10\rangle \mapsto |11\rangle$$

$$|11\rangle \mapsto |10\rangle$$

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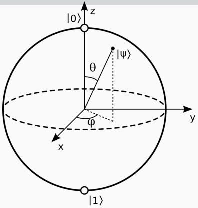

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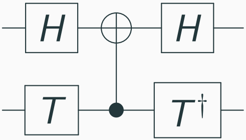

$$|00\rangle \mapsto |00\rangle$$
$$|01\rangle \mapsto |01\rangle$$
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$$T \equiv \mathcal{R}_{\pi/4}$$

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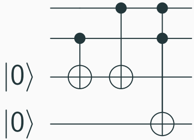
$$\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix} \qquad \xcancel{\begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix}}$$

Classical Algorithms Are Quantum Algorithms

Any classical circuit can be made into a quantum circuit!

Example: two-bit adder

$$|00\rangle\,|00\rangle \to |00\rangle\,|00\rangle$$
$$|01\rangle\,|00\rangle \to |01\rangle\,|01\rangle$$
$$|10\rangle\,|00\rangle \to |10\rangle\,|01\rangle$$
$$|11\rangle\,|00\rangle \to |11\rangle\,|10\rangle$$

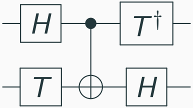


In general, given a classical function $f(x)$, we can implement:

$$|x\rangle\,|0\rangle \to |x\rangle\,|f(x)\rangle$$

Inverting Circuits

$$(AB)^{-1} = B^{-1}A^{-1}$$



As long as we have the inverse of each individual gate...

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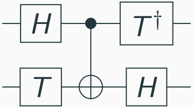

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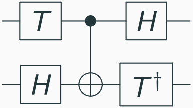
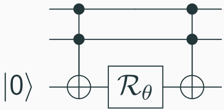


Phase Gates

Easy version of real-time evolution: $\mathcal{U}(t) = e^{-iHt}$, with H diagonal.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{-i\theta} \end{pmatrix}$$

$$|11\rangle \to e^{-i\theta} |11\rangle$$



Real-Time Evolution

Goal: e^{-iHt}

$$e^{-i(H_1+H_2)\epsilon} \approx e^{-iH_1\epsilon} e^{-iH_2\epsilon}$$

1. Split H into tiny pieces
2. Diagonalize each piece
3. Hit with phase gates

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On a classical computer: $2^N \times 2^N$ matrices.

On a quantum computer: N qubits and $\propto t/\epsilon$ gates.

Example: Coupled Spins

$$H = \overbrace{\sigma_z(1)\sigma_z(2)}^{H_V} + \overbrace{\mu\left(\sigma_x(1) + \sigma_x(2)\right)}^{H_K}$$

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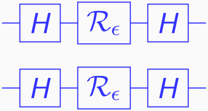
$$e^{-iHt} \approx \left(e^{-iH_V\epsilon}e^{-iH_K\epsilon}\right)^{t/\epsilon}$$

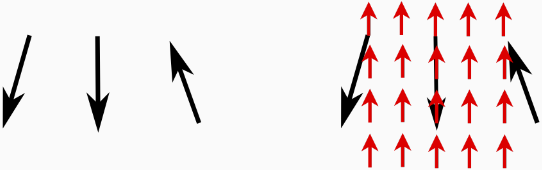
H_V is diagonal!

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\epsilon} & 0 & 0 \\ 0 & 0 & e^{i\epsilon} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

H_K is diagonalized by the Hadamard gate.

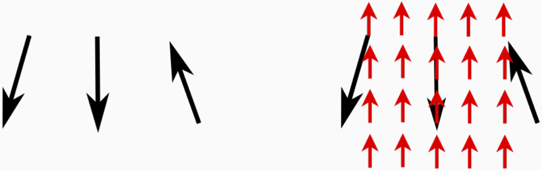


Real-Time Evolution for Nonlinear Response



$$|\Psi\rangle \rightarrow e^{-iH_0 T}|\Psi\rangle \rightarrow e^{-i(H_0+H_B)t}e^{-iH_0 T}|\Psi\rangle \rightarrow \text{Measure!}$$

Real-Time Evolution for Nonlinear Response



$$|\Psi\rangle \rightarrow e^{-iH_0 T}|\Psi\rangle \rightarrow e^{-i(H_0+H_B)t}e^{-iH_0 T}|\Psi\rangle \rightarrow \text{Measure!}$$

This gives $\langle \mathcal{O}(t) \rangle$. What about $\langle \mathcal{O}(t)\mathcal{O}(0) \rangle$?

Want linear response? **Take a derivative!**

Real-Time Evolution for Linear Response

Want linear response? **Take a derivative!**

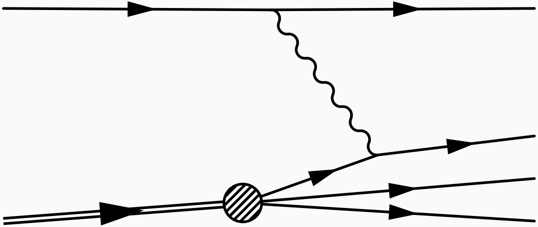
$$H(t) = H_0 + \epsilon\delta(t)H'$$

1. Evolve briefly with $H' = \mathcal{O}$. $e^{-i\mathcal{O}\epsilon}\left|\Psi\right\rangle$
2. Perform normal time-evolution. $e^{-iH_0 t}e^{-i\mathcal{O}\epsilon}\left|\Psi\right\rangle$
3. Measure $\mathcal{O}$.

$$\frac{\partial}{\partial\epsilon}\left\langle e^{iHt}\mathcal{O}e^{-iHt}\right\rangle = \mathrm{Im}\left\langle\mathcal{O}(t)\mathcal{O}(0)\right\rangle$$

Alternatives include: Roggero and Carlson `1804.01505`; Pedernales et al. `1401.2430`.

Parton Distribution Functions



$$f_q(\xi) = \int_{\infty}^{\infty} \frac{\mathrm{d}t}{2\pi} e^{-it\xi(n\cdot P)} \langle P| \bar{\psi}_q(tn^{\mu}) \frac{\not{n}}{2} W_n \psi_q(0) |P\rangle$$

Preparing the Ground State

Naive method: couple to a large thermal bath.

$$H = H_0 + H_{\mathrm{bath}} + H_{\mathrm{int}}$$

If H_{bath} is well-understood (easily arranged), we can prepare it cold, and then time-evolve.

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If H_{bath} is well-understood (easily arranged), we can prepare it cold, and then time-evolve.

More sophisticated: Spectral Combing (`1709.08250`), Quantum Adiabatic Algorithm (`quant-ph/0001106`) both require e^{-iHt}.

Hybrid classical/quantum methods don't: `1806.06649`.

Quantum Mechanics on a Group

The Hamiltonian of a free particle moving on $G = SU(3)$:

$$H = -\nabla^2$$

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The Hilbert space is $\mathbb{C}G$: one basis state for every $U \in G$.

$$\left|\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right\rangle, \left|\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & i \\ -i & 0 & 0 \end{pmatrix}\right\rangle, \left|\begin{pmatrix} 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{pmatrix}\right\rangle$$

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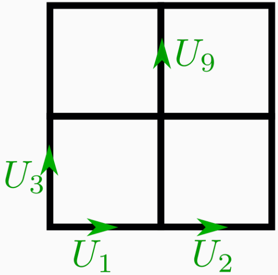
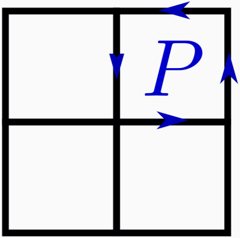
The Hamiltonian is diagonal in Fourier space. (For R, momentum space.)

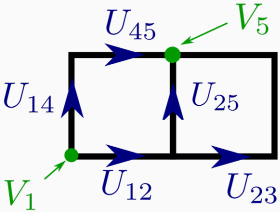
Hamiltonian Lattice Gauge Theory





$$H = \frac{1}{g^2}\left[\sum_L \nabla_L^2 + \sum_P \mathrm{Re}\,\mathrm{Tr}\,P\right]$$

Gauge Symmetry



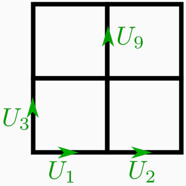
$$U_{ij} \mapsto V_j U_{ij} V_i^\dagger$$

$$\begin{aligned} &\mathrm{Tr}\, U_{14}^\dagger U_{45}^\dagger U_{25} U_{12} \\ \mapsto &\mathrm{Tr}\, U_{14}^\dagger U_{45}^\dagger U_{25} V_2^\dagger V_2 U_{12} \end{aligned}$$

The Hamiltonian is gauge-invariant

$$H = \frac{1}{g^2} \left[\sum_{\langle ij \rangle} \nabla_{ij}^2 + \sum_P \mathrm{Re}\,\mathrm{Tr}\, P \right]$$

The Hilbert Space



$$\mathcal{H} = \mathbb{C}G \otimes \mathbb{C}G \otimes \cdots$$

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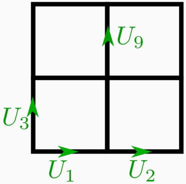

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Only Gauge-Invariant States Allowed!

$$|U_{12}\rangle \qquad \int \mathrm{d}V_1 \mathrm{d}V_2 \left| V_2^\dagger U_{12} V_1 \right\rangle$$

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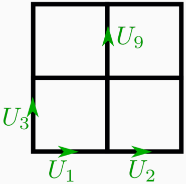

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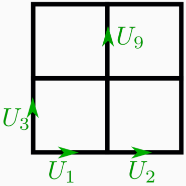

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Only Gauge-Invariant States Allowed!

$$\cancel{|U_{12}\rangle} \qquad \int \mathrm{d}V_1 \mathrm{d}V_2 \left| V_2^\dagger U_{12} V_1 \right\rangle$$

Here's a projection operator:

$$P\,|U_{12}\cdots\rangle = \int (\mathrm{d}V_1 \mathrm{d}V_2 \cdots) \left| V_2^\dagger U_{12} V_1 \cdots \right\rangle$$

Trotterization

$$H = \frac{1}{g^2}\left[\overbrace{\sum_L \nabla_L^2}^{H_K} + \overbrace{\sum_P \operatorname{Re}\operatorname{Tr} P}^{H_V}\right]$$

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Kinetic	**Potential**
One link only	Four links
Diagonal in Fourier space	Diagonal (in our basis)
Mutually commuting terms	Mutually commuting terms

$$e^{-iHt} \approx \left[\left(e^{-i\nabla_1^2\epsilon} e^{-i\nabla_2^2\epsilon}\dots\right)\left(e^{-i\epsilon \operatorname{Re}\operatorname{Tr} P_1} e^{-i\epsilon \operatorname{Re}\operatorname{Tr} P_2}\dots\right)\right]^{t/\epsilon}$$

Hilbert Space on a Quantum Computer

Classical algorithms ⟶ quantum algorithms!

$$SU(3) \longleftrightarrow \{0000, 0001, 0010, \cdots\}$$

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That's a basis for $\tilde{\mathcal{H}}_1$! An "$SU(3)$-register" — the analog of a classical variable.

Now that we have $\mathcal{H}_1 \sim \tilde{\mathcal{H}}_1$, we can construct the full $\mathcal{H}$ on a quantum computer.

$$|U_{12}\rangle |U_{23}\rangle \cdots \in \tilde{\mathcal{H}}_1 \otimes \tilde{\mathcal{H}}_1 \otimes \cdots$$

The set of physical states is a linear subspace.

Potential Term

$$H = \frac{1}{g^2}\left[\sum_L \nabla_L^2 + \overbrace{\sum_P \mathrm{Re}\,\mathrm{Tr}\,P}^{H_V}\right]$$

We need an operator:

$$\mathcal{U}(\theta)\,|A\rangle\,|B\rangle\,|C\rangle\,|D\rangle = e^{-i\theta\,\mathrm{Re}\,\mathrm{Tr}(ABCD)}\,|A\rangle\,|B\rangle\,|C\rangle\,|D\rangle$$

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$$\left|A\right\rangle\left|B\right\rangle\left|C\right\rangle\left|D\right\rangle$$

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$$\begin{aligned} &\left|A\right\rangle\left|B\right\rangle\left|C\right\rangle\left|D\right\rangle \\ \rightarrow &\left|A\right\rangle\left|B\right\rangle\left|C\right\rangle\left|CD\right\rangle \end{aligned}$$

Potential Term

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\to \cdots &\to |A\rangle\,|B\rangle\,|C\rangle\,|ABCD\rangle\\
&\to |A\rangle\,|B\rangle\,|C\rangle\,e^{-i\theta\,\mathrm{Re}\,\mathrm{Tr}(ABCD)}\,|ABCD\rangle
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\end{aligned}$$

Kinetic Term

$$H = \frac{1}{g^2}\left[\overbrace{\sum_L \nabla_L^2}^{H_K} + \sum_P \operatorname{Re}\operatorname{Tr} P\right]$$

Diagonal in "momentum basis". Need to perform a Quantum Fourier Transform. This is a Fourier transform of $\Psi(x)$:

$$|\Psi\rangle = \sum_x \Psi(x)\,|x\rangle$$

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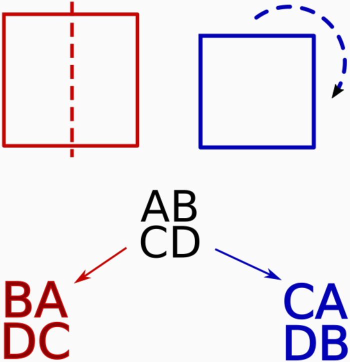
For $\mathbb{R}$:

$$F\,|x\rangle = \sum_p e^{-ixp}\,|p\rangle$$

For $SO(3)$: QFT decomposes into spherical harmonics

So: QFT, then phase gate (on a single link!), then QFT.

The Dihedral Group D_4



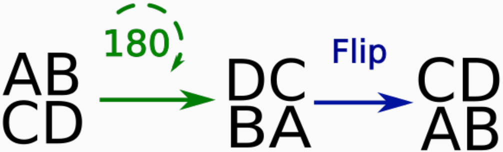
As a matrix group, $D_4 < U(2)$.

$$\left\langle \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \right\rangle$$

The D_4-Register

$|D_4| = 8$, so we want 3 qubits.

$$\underbrace{1}_{\text{Then flip}}\overbrace{10}^{\text{Rotate twice}}$$



Inversion

Classical circuits ⟶ quantum circuits!

$|000\rangle \to |000\rangle$

$|001\rangle \to |011\rangle$

$|010\rangle \to |010\rangle$

$|011\rangle \to |001\rangle$

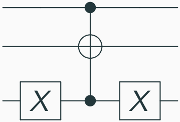
$|100\rangle \to |100\rangle$

$|101\rangle \to |101\rangle$

$|110\rangle \to |110\rangle$

$|111\rangle \to |111\rangle$

$$X = \begin{pmatrix} 0 & 1 & \\ & & |0\rangle \leftrightarrow |1\rangle \\ 1 & 0 & \end{pmatrix}$$



Multiplication

Classical circuits ⟶ quantum circuits!

$$\mathcal{U}_\times |U\rangle |V\rangle = |U\rangle |UV\rangle$$

Because G is a group, this operation is unitary.

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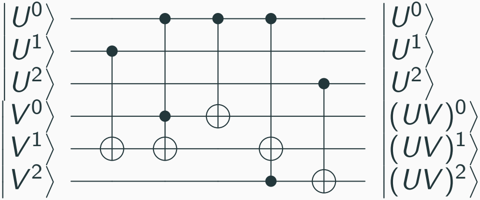

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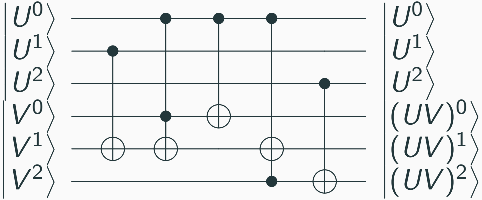


Multiplication and inversion let us construct a plaquette:

$$\mathcal{U}_P |U_{12} \cdots\rangle |\mathbf{1}\rangle = |U_{12} \cdots\rangle |P\rangle$$

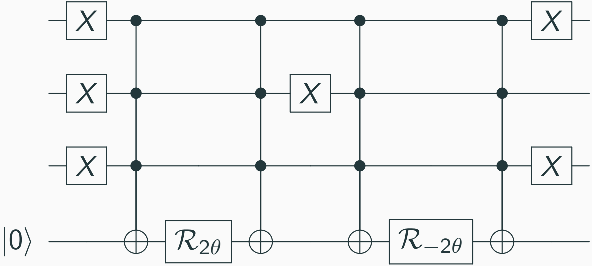
Trace

$$\mathcal{U}_{\mathsf{Tr}}(\theta)\,|U\rangle = e^{i\theta\,\mathrm{Re}\,\mathrm{Tr}\,U}\,|U\rangle$$

Only two elements of D_4 have non-zero trace.

$$\mathrm{Tr}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = 2 \qquad\qquad \mathrm{Tr}\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix} = -2$$

These correspond to the states $|000\rangle$ and $|010\rangle$.



Fourier Transform

$$\begin{pmatrix}
\frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} \\
\frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{-1}{\sqrt{8}} & \frac{-1}{\sqrt{8}} & \frac{-1}{\sqrt{8}} & \frac{-1}{\sqrt{8}} \\
\frac{1}{\sqrt{8}} & -\frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & -\frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & -\frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & -\frac{1}{\sqrt{8}} \\
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\frac{1}{2} & 0.0 & -\frac{1}{2} & 0.0 & \frac{1}{2} & 0.0 & -\frac{1}{2} & 0.0 \\
0.0 & -\frac{1}{2} & 0.0 & \frac{1}{2} & 0.0 & \frac{1}{2} & 0.0 & -\frac{1}{2} \\
0.0 & \frac{1}{2} & 0.0 & -\frac{1}{2} & 0.0 & \frac{1}{2} & 0.0 & -\frac{1}{2} \\
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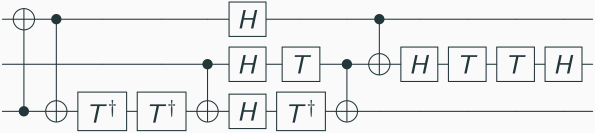
\end{pmatrix}$$

Yikes! **This is a job for a computer...**



The **global** Hilbert space is large, and can't be treated classically.

The **local** Hilbert space is small, so **it can be treated classically**.

Kinetic Term

The diagonalized kinetic operator is

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-4it} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-4it} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & e^{-4it} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{-4it} \end{pmatrix} \begin{pmatrix} |000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \\ |110\rangle \\ |111\rangle \end{pmatrix}$$

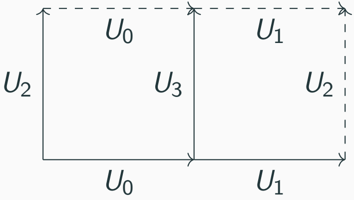
Overview of the Simulation

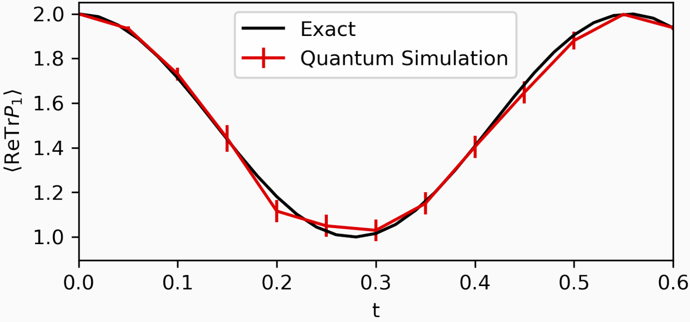
$\mathcal{H} = \mathbb{C}D_4 \otimes \mathbb{C}D_4 \otimes \cdots$ maps onto $3 \times L$ qubits

1. Prepare initial state (somehow)
2. Trace circuit on all plaquettes
3. QFT on all links
4. Phase gate on the most-significant qubit of all links
5. $\text{QFT}^\dagger$ on all links
6. Repeat 2-5 to get the desired t
7. Measure; look at something gauge-invariant

Requires $\sim$ 300 gates per time step.

Real-Time Evolution





A Comment on Trotterization

Deriving time evolution

$$
\begin{aligned}
U &= e^{-iHt} \\
&= \left(e^{-i(H_1+H_2)\delta}\right)^{t/\delta} \\
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It's the same procedure!

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It's the same procedure!

Physics on the classical lattice should be similar to physics on the quantum lattice.

Coupling Constants on the Cheap!

$$H = \frac{1}{g^2}\left[a^3\sum_{\langle ij\rangle}\nabla_{ij}^2 + a^2\sum_{P}\mathrm{Re}\,\mathrm{Tr}\,P\right] + a^3 m\sum_{i}\bar{\psi}_i\psi_i + a^2\sum_{\langle ij\rangle}\bar{\psi}_i\psi_j$$

We want $m_\pi \approx 135\mathrm{MeV}$. What should a, m, and g be?

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We want $m_\pi \approx 135\mathrm{MeV}$. What should a, m, and g be?

Use the euclidean lattice!

Spatial discretization and finite volume effects are the same: desired couplings can be found classically.

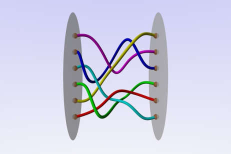
The Present

The Near Future

Current best

$\sim$ 10 qubits

10 gates



“Quantum supremacy”

50 qubits

40 gates

Needed (for 10^3 lattice)

$$\overbrace{10^3 \times 3}^{\text{links}} \times \overbrace{16 \times 18}^{SU(3)\text{-register}} \sim 10^6 \text{ qubits}$$

Why bother?

Concrete staring point for more efficient algorithms

Are large-scale quantum processors worth building?

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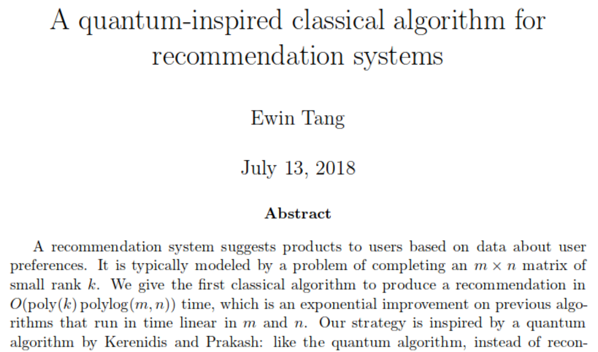
Concrete staring point for more efficient algorithms

Are large-scale quantum processors worth building?

Inspiration for new classical algorithms



A quantum-inspired classical algorithm for recommendation systems

Ewin Tang

July 13, 2018

Abstract

A recommendation system suggests products to users based on data about user preferences. It is typically modeled by a problem of completing an $m \times n$ matrix of small rank k. We give the first classical algorithm to produce a recommendation in $O(\text{poly}(k)\,\text{polylog}(m, n))$ time, which is an exponential improvement on previous algorithms that run in time linear in m and n. Our strategy is inspired by a quantum algorithm by Kerenidis and Prakash: like the quantum algorithm, instead of recon-

What don’t we know?

- How few qubits can we get away with?
- Can error correction/tolerance be done more cheaply?
- How to prepare QCD ground state?
- What algorithms *don’t* involve e^{-iHt}?

Outline

Simulating QCD

Overview of Quantum Computing

Straightforward Algorithms

A Crucial Building Block

Simulating Gauge Theories

Bits and Pieces — D_4

Simulated Simulations

Choosing Coupling

Future Work