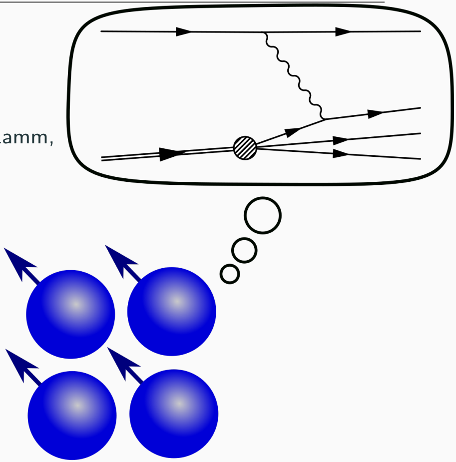
From Qubits to Quarks:
Parton Physics on a Quantum Computer

Scott Lawrence
with Andrei Alexandru, Paulo Bedaque,
Siddhartha Harmalkar, Hersh Kumar, Henry Lamm,
Neill Warrington, and Yukari Yamauchi

1903.08807, 1906.11213, 1908.10439

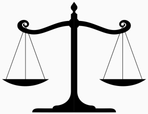
20 December 2019

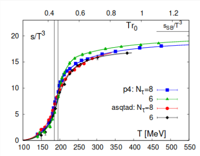
UNIVERSITY OF MARYLAND



Some Questions in Nonperturbative QCD

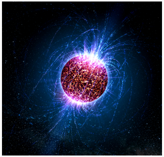
Masses

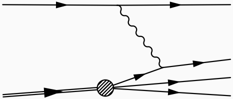




Finite temperature equation of state

Cold, dense equation of state





Parton Physics

0903.4379

Parton Physics on the Eucidean Lattice

$$f(x) = \int \mathrm{d}z \; e^{ixP^+z} \langle P| \, e^{iHz} \bar{\psi}(z) e^{-iHz} \gamma^+ \psi(0) \, |P\rangle$$

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Many methods available: Quasi PDFs, Pseudo PDFs...

Parton Physics on Euclidean Lattice

Xiangdong Ji[1,2]

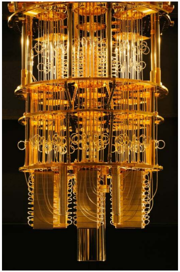
Exploring quark transverse momentum distributions with lattice QCD

B.U. Musch,[1] Ph. Hägler,[2,3] J.W. Negele,[4] and A. Schäfer[3]

Complexities stem from Euclidean $\longrightarrow$ Minkowski.

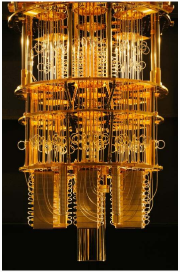
PDFs are *natural* in a quantum simulation.

The Great Quantum Hope



50 qubit machines now available.

The Great Quantum Hope



50 qubit machines now available.

Quantum supremacy using a programmable superconducting processor

Google AI Quantum and collaborators[†]

The tantalizing promise of quantum computers is that certain computational tasks might be executed exponentially faster on a quantum processor than on a classical processor. A fundamental challenge is to build a high-fidelity processor capable of running quantum algorithms in an exponentially large computational space. Here, we report using a processor with programmable superconducting qubits to create quantum states on 53 qubits, occupying a state space $2^{53} \sim 10^{16}$.

Towards analog quantum simulations of lattice gauge theories with trapped ions

Zohreh Davoudi,[1,2,*] Mohammad Hafezi,[3,4] Christopher Monroe,[3,5] Guido Pagano,[3,5] Alireza Seif,[3] and Andrew Shaw[1]

Realistically...

Current quantum processors: 50 qubits, 10 gates

Are large processors worth it?

Realistically...

Current quantum processors: 50 qubits, 10 gates

Are large processors worth it?

A quantum-inspired classical algorithm for recommendation systems

Ewin Tang

July 13, 2018

Abstract

A recommendation system suggests products to users based on data about user preferences. It is typically modeled by a problem of completing an $m \times n$ matrix of small rank k. We give the first classical algorithm to produce a recommendation in $O(\text{poly}(k)\,\text{polylog}(m,n))$ time, which is an exponential improvement on previous algorithms that run in time linear in m and n. Our strategy is inspired by a quantum algorithm by Kerenidis and Prakash: like the quantum algorithm, instead of recon-

Quantum-inspired classical algorithms for principal component analysis and supervised clustering

Ewin Tang

November 2, 2018

Abstract

We describe classical analogues to quantum algorithms for principal component analysis and nearest-centroid clustering. Given sampling assumptions, our classical algorithms run in time polylogarithmic in input, matching the runtime of the quantum algorithms with only polynomial slowdown. These algorithms are evidence that their corresponding problems do not yield exponential quantum speedups. To build our

Overview

A quantum computer is a quantum system evolved in real-time

QC Hilbert space $\Longleftrightarrow$ physical Hilbert space

Parton distribution function:

$$f(x) = \int \mathrm{d}z \; e^{ixP^+ z} \langle P | \, e^{iHz} \bar{\psi}(z) e^{-iHz} \gamma^+ W \psi(0) \, | P \rangle$$

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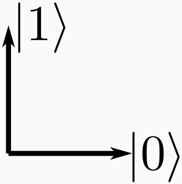
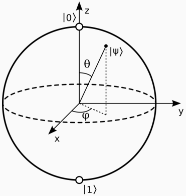
- Some quantum algorithms
- Hamiltonian formulation of gauge theory
- Putting $SU(3)$ on a quantum computer: $S(1080) < SU(3)$
- Parton physics of $S(1080)$ gauge theory

From Bit to Qubit

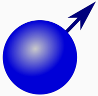
$$|0\rangle\,,\ \ |1\rangle\ldots$$

$$|0\rangle\,,\ |1\rangle \ldots\ \frac{i}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle$$

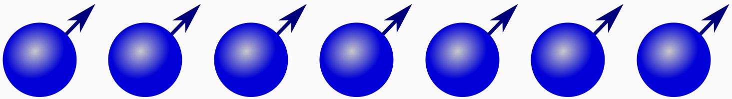




This is a spin from QM.



A Quantum Computer



Physically, there's a Hilbert space:

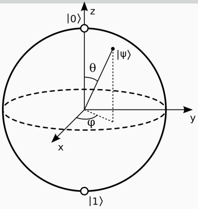
$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_1 \otimes \cdots$$

When we measure, we collapse into one of the 2^N states in the "fiducial basis".

$$|\Psi\rangle \rightarrow |0101010\rangle$$

Gates

Arbitrary one-qubit gates are 'easy' – can be constructed from Hadamard and $\frac{\pi}{8}$-gate.



$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \; T = \begin{pmatrix} e^{i\pi/8} & 0 \\ 0 & e^{-i\pi/8} \end{pmatrix}$$

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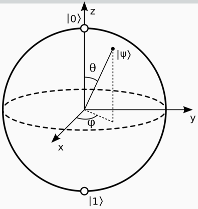

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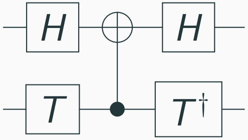
Controlled-not (CNOT) is a 2-qubit gate.



$$|00\rangle \mapsto |00\rangle$$
$$|01\rangle \mapsto |01\rangle$$
$$|10\rangle \mapsto |11\rangle$$
$$|11\rangle \mapsto |10\rangle$$

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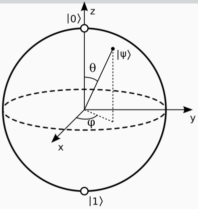

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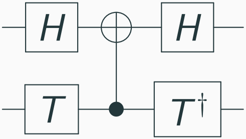

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$$T \equiv \mathcal{R}_Z(\pi/4)$$

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A quantum computer constantly being measured in the fiducial basis.

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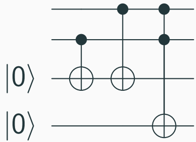
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$$

Classical Algorithms Are Quantum Algorithms

Any classical circuit can be made into a quantum circuit!

Example: two-bit adder

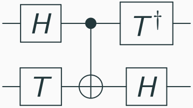
$$|00\rangle\,|00\rangle \rightarrow |00\rangle\,|00\rangle$$
$$|01\rangle\,|00\rangle \rightarrow |01\rangle\,|01\rangle$$
$$|10\rangle\,|00\rangle \rightarrow |10\rangle\,|01\rangle$$
$$|11\rangle\,|00\rangle \rightarrow |11\rangle\,|10\rangle$$



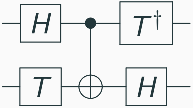
In general, given a classical function $f(x)$, we can implement:

$$|x\rangle\,|0\rangle \rightarrow |x\rangle\,|f(x)\rangle$$

Inverting Circuits



As long as we have the inverse of each individual gate...



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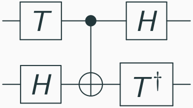
$$(AB)^{-1} = B^{-1}A^{-1}$$

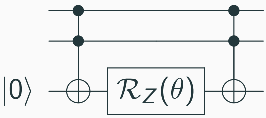


Phase Gates

Easy version of real-time evolution: $\mathcal{U}(t) = e^{-iHt}$, with H diagonal.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{-i\theta} \end{pmatrix}$$

$$|11\rangle \rightarrow e^{-i\theta} |11\rangle$$



Real-Time Evolution (Trotter-Suzuki)

Goal: e^{-iHt}

$$e^{-i(H_1+H_2)\epsilon} \approx e^{-iH_1\epsilon}e^{-iH_2\epsilon}$$

1. Split H into tiny pieces
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On a classical computer: $2^N \times 2^N$ matrices.

On a quantum computer: N qubits and $\propto t/\epsilon$ gates.

Example: Coupled Spins

$$H = \overbrace{\sigma_z(1)\sigma_z(2)}^{H_V} + \overbrace{\mu\left(\sigma_x(1) + \sigma_x(2)\right)}^{H_K}$$

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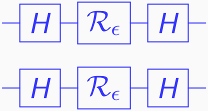
$$e^{-iHt} \approx \left(e^{-iH_V\epsilon}e^{-iH_K\epsilon}\right)^{t/\epsilon}$$

H_V is diagonal!

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\epsilon} & 0 & 0 \\ 0 & 0 & e^{i\epsilon} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

H_K is diagonalized by the Hadamard gate.



Measurements

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle = \langle \Psi | e^{iHt}\mathcal{O}e^{-iHt}\mathcal{O} | \Psi \rangle$$

This is not a Hermitian operator!

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Perturb the Hamiltonian:

$$H'_\epsilon(t) = H + \epsilon\delta(t)\mathcal{O}$$

And now estimate the derivative:

$$\mathrm{Im}\,\langle\Psi|\mathcal{O}(t)\mathcal{O}(0)|\Psi\rangle = \frac{1}{2}\frac{\partial}{\partial\epsilon}\langle\Psi| e^{i\mathcal{O}\epsilon}e^{iHt}\mathcal{O}e^{-iHt}e^{-i\mathcal{O}\epsilon}|\Psi\rangle$$

Quantum Mechanics on a Group

The Hamiltonian of a free particle moving on $G = SU(3)$:

$$H = -\nabla^2$$

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The Hilbert space is $\mathbb{C}G$: one basis state for every $U \in G$.

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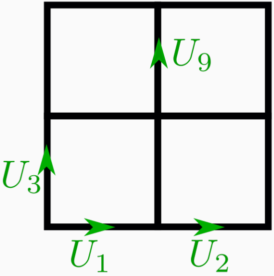
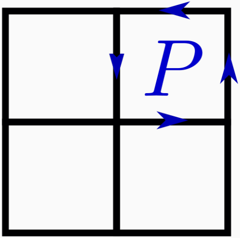
The Hamiltonian is diagonal in Fourier space. (For $\mathbb{R}$, momentum space.)

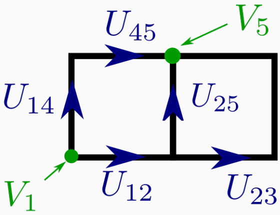
Hamiltonian Lattice Gauge Theory





$$H = \beta_K \sum_L \nabla_L^2 + \beta_P \sum_P \mathrm{Re}\,\mathrm{Tr}\,P$$

Gauge Symmetry



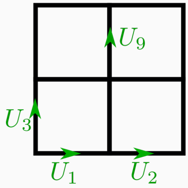
$$U_{ij} \mapsto V_j U_{ij} V_i^\dagger$$

$$\begin{aligned}&\operatorname{Tr} U_{14}^\dagger U_{45}^\dagger U_{25} U_{12} \\ \mapsto &\operatorname{Tr} U_{14}^\dagger U_{45}^\dagger U_{25} V_2^\dagger V_2 U_{12}\end{aligned}$$

The Hamiltonian is gauge-invariant

$$H = \beta_K \sum_{\langle ij \rangle} \nabla_{ij}^2 + \beta_P \sum_P \operatorname{Re} \operatorname{Tr} P$$

The Hilbert Space



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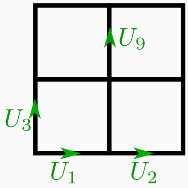

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Only Gauge-Invariant States Allowed!

$$|U_{12}\rangle \qquad\qquad \int \mathrm{d}V_1 \mathrm{d}V_2 \left| V_2^\dagger U_{12} V_1 \right\rangle$$

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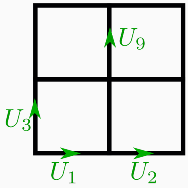

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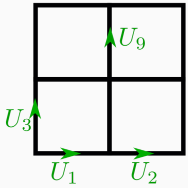

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Only Gauge-Invariant States Allowed!

~~$|U_{12}\rangle$~~

$$\int \mathrm{d}V_1 \mathrm{d}V_2 \left| V_2^\dagger U_{12} V_1 \right\rangle$$

Here's a projection operator:

$$P\,|U_{12}\cdots\rangle = \int (\mathrm{d}V_1 \mathrm{d}V_2 \cdots) \left| V_2^\dagger U_{12} V_1 \cdots \right\rangle$$

Trotterization

$$H = \frac{1}{g^2}\left[\overbrace{\sum_L \nabla_L^2}^{H_K} + \overbrace{\sum_P \mathrm{Re}\,\mathrm{Tr}\,P}^{H_V}\right]$$

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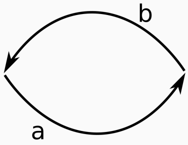
Kinetic	**Potential**
One link only	Four links
Diagonal in Fourier space	Diagonal (in our basis)
Mutually commuting terms	Mutually commuting terms

$$e^{-iHt} \approx \left[\left(e^{-i\nabla_1^2\epsilon}e^{-i\nabla_2^2\epsilon}\cdots\right)\left(e^{-i\epsilon\,\mathrm{Re}\,\mathrm{Tr}\,P_1}e^{-i\epsilon\,\mathrm{Re}\,\mathrm{Tr}\,P_2}\cdots\right)\right]^{t/\epsilon}$$

Example: $\mathbb{Z}_2$, One 'Plaquette'

$$H = \sigma_x(a) + \sigma_x(b) + \sigma_z(a)\sigma_z(b)$$



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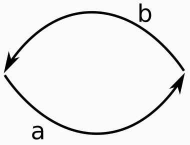

The gauge transformation operator: $\sigma_x(a)\sigma_x(b)$.

$$|00\rangle \leftrightarrow |11\rangle \text{ and } |01\rangle \leftrightarrow |10\rangle$$

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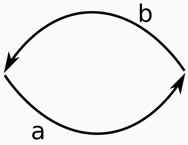

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Physical states: $|P=0\rangle = |00\rangle + |11\rangle$, $|P=1\rangle = |01\rangle + |10\rangle$

Unphysical states: $|00\rangle - |11\rangle$, $|01\rangle - |10\rangle$

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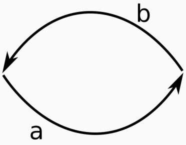
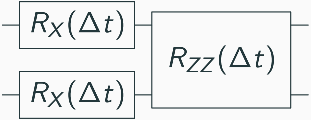
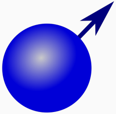


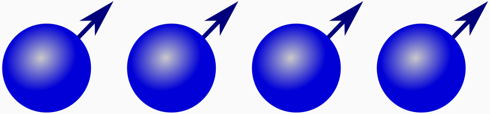


The Problem with $SU(3)$

$SU(3)$ mechanics has an infinite dimensional Hilbert space.



One qubit has a 2-dimensional Hilbert space.



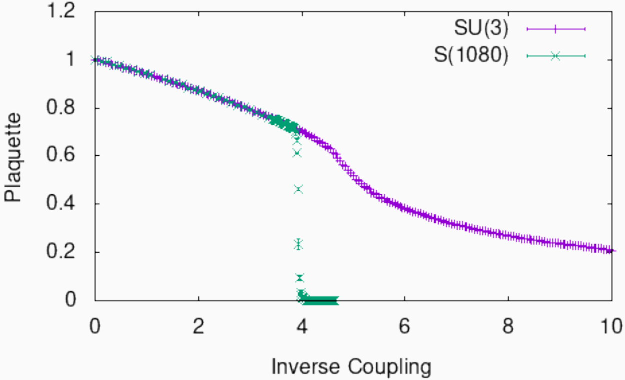
$$2^N < \infty$$

Any implementation of $SU(3)$ on a QC **must be an approximation.**

A Natural Truncation

Largest 'nice' finite subgroup of $SU(3)$: $S(1080)$

Simulate $S(1080)$ gauge theory instead



Does it work?

With an improved action:

$$S = -\sum_p \left(\frac{\beta_0}{3} \operatorname{Re}\operatorname{Tr} U_p + \beta_1 \operatorname{Re}\operatorname{Tr} U_p^2 \right)$$

Measure two scales, and compare the ratio to $SU(3)$: Wilson flow, center symmetry

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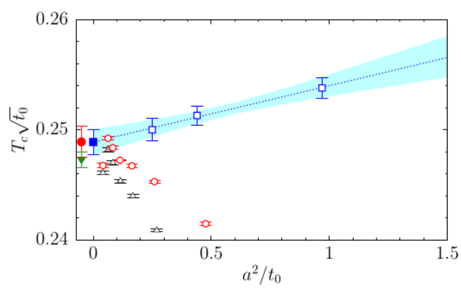

T_c: center symmetry breaking

t_0: solution to $0.3 = t_0^2 \langle E \rangle_{t_0}$

Dimensionless ratio $T_c\sqrt{t_0}$.

1906.11213

Does it actually work?

Smallest lattice spacing is $a = 0.08$ fm.

At this spacing, $S(1080)$ and $SU(3)$ agree on the low-energy observable $T_c\sqrt{t_0}$.

Beyond this spacing, they disagree violently.

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Do other low-energy quantities agree?

In progress: spectroscopy, further improved actions

So what do we need?

- Map from $S(1080)$ Hilbert space to the quantum computer
- Time evolution: kinetic and potential pieces.
- State preparation
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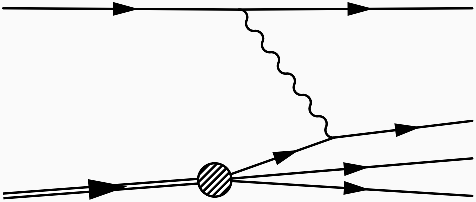

$S(1080)$ Hilbert Space on a Quantum Computer

Classical algorithms $\longrightarrow$ quantum algorithms!

$$S(1080) \longleftrightarrow \{\ldots 000, \ldots 001, \ldots 010, \cdots\}$$

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That's a basis for $\mathcal{H}_1$! An "$S(1080)$-register" — the analog of a classical variable.

Now that we have $\mathcal{H}_1$, we can construct the full $\mathcal{H}$ on a quantum computer.

$$|U_{12}\rangle |U_{23}\rangle \cdots \in \tilde{\mathcal{H}}_1 \otimes \tilde{\mathcal{H}}_1 \otimes \cdots$$

The set of physical states is a linear subspace.

Potential Term

$$H = \frac{1}{g^2}\left[\sum_L \nabla_L^2 + \overbrace{\sum_P \operatorname{Re}\operatorname{Tr} P}^{H_V}\right]$$

We need an operator:

$$\mathcal{U}(\theta)\,|A\rangle\,|B\rangle\,|C\rangle\,|D\rangle = e^{-i\theta \operatorname{Re}\operatorname{Tr}(ABCD)}\,|A\rangle\,|B\rangle\,|C\rangle\,|D\rangle$$

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Diagonal in "momentum basis". Need to perform a Quantum Fourier Transform. This is a Fourier transform of $\Psi(x)$:

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For $\mathbb{R}$:

$$F\,|x\rangle = \sum_p e^{-ixp}\,|p\rangle$$

For $SO(3)$: QFT decomposes into spherical harmonics

So: QFT, then phase gate (on a single link!), then QFT.

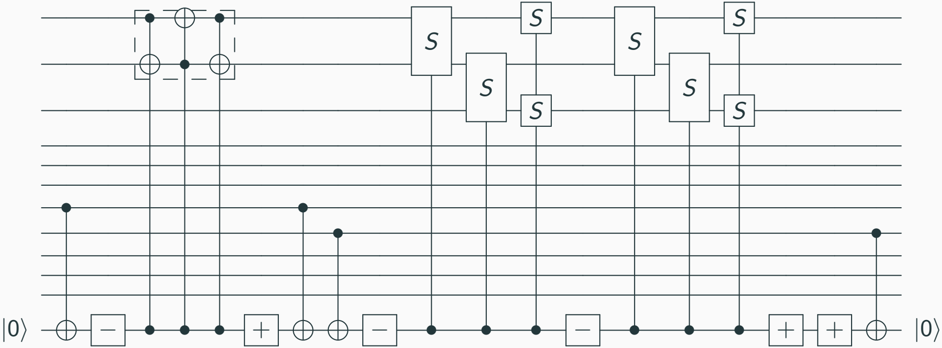
Bits and Pieces

Primitive gates used:

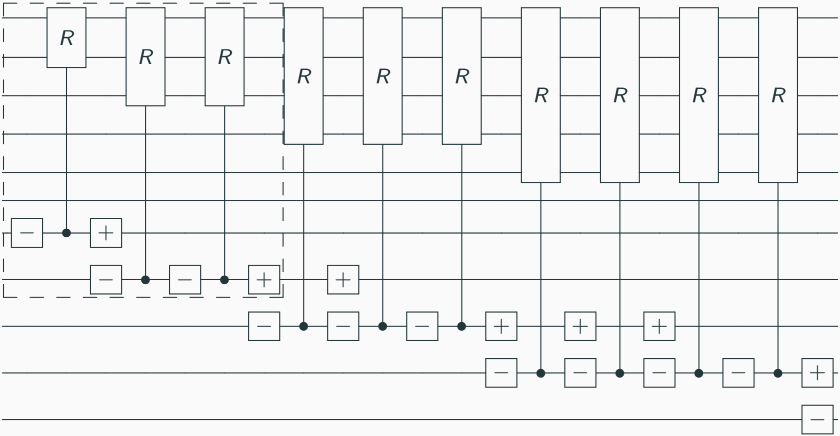
- Multiplication
- Inversion
- Trace gate (a diagonal rotation)
- Fourier transform
- Kinetic gate (a diagonal rotation)

Gate construction for $S(1080)$ in progress!

Multiplication for $S(1080)/\mathbb{Z}_3$



Multiplication for $S(1080)/\mathbb{Z}_3$ (More Schematic)



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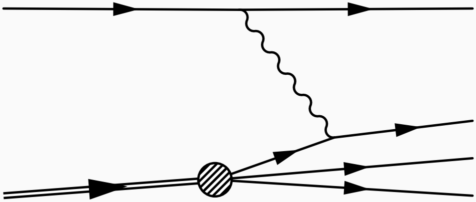

State preparation: A thousand flowers...

- Thermal bath
- Adiabatic state preparation
- Spectral comb (1709.08250)
- Hybrid state preparation (**PhysRevLett 121 170501**, 1908.07051)

Nobody knows how most of these things scale.

And we certainly can't test them on large systems *now*

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Formal guarantees available for adiabatic state preparation

Adiabatic theorem

Take a time-varying (slowly) Hamiltonian $H(t)$.

Prepare an eigenstate of $H(0)$, with a gap of Δ.

When $\dot{H}/\Delta^2 \ll 0$, time-evolution will keep us in the eigenstate.

Time needed to prepare ground state: Δ^{-2}

Preparing the proton

Restrict to a certain sector of Hilbert space:

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Restrict to a certain sector of Hilbert space:

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- Free fermions and glue (massive)
- Ground state exactly prepared
- Small gap ($O(\frac{1}{V})$)

- Hadrons
- Large gap (m_π)

Total circuit size: $O(V^3)$

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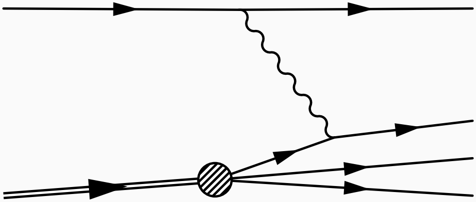

Thirring PDF

$$H = \int \mathrm{d}x \, \bar{\psi} \left(\not{\partial} + m \right) \psi + g^2 \left(\bar{\psi} \psi \right)^2$$

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Staggered discretization:

$$H = \sum_r \frac{1}{2}(-1)^r \left(\chi_r^\dagger \chi_{r+1} + \chi_{r+1}^\dagger \chi_r \right) + m(-1)^r \chi_r^\dagger \chi_r - g^2 \chi_r^\dagger \chi_r \chi_{r+1}^\dagger \chi_{r+1}$$

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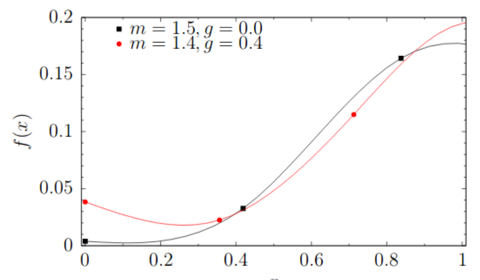
Parton distribution function:

$$f(x) = \int \mathrm{d}z\, e^{ixP^+z}\langle P|\, e^{iHz}\bar{\psi}(z)e^{-iHz}\gamma^+\psi(0)\,|P\rangle$$



Some (relatively minor) PDF complexities

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Convergence of the Fourier transform

$$\int \mathrm{d}z \; e^{ikz} C(z) \equiv \lim_{\epsilon \to 0} \lim_{L \to \infty} \int_{-L}^{L} \mathrm{d}z \; e^{ikz} e^{-\epsilon z^2} C(z)$$

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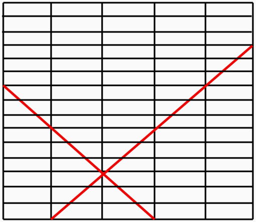
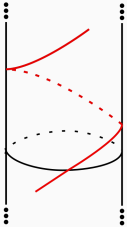
Light cone: there really is none on the lattice.





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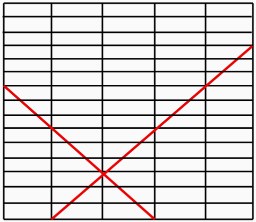

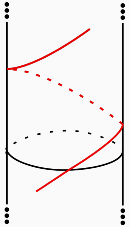

None of which matters compared to...

A long line...

$$\left\langle \bar{\psi}(x)\gamma^{+} W(x;0)\psi(0) \right\rangle_{\mathrm{proton}}$$

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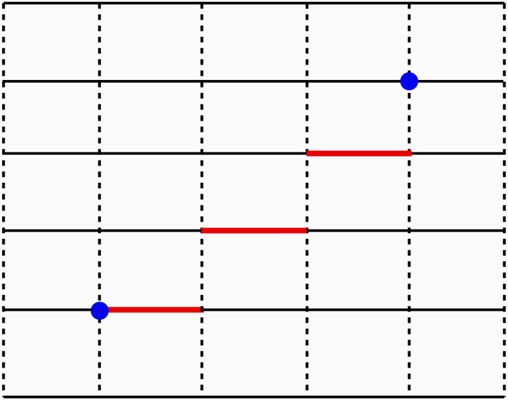

The Hadronic Tensor

$$W^{\mu\nu}(q) = \int \mathrm{d}x \; e^{iqx} \left\langle e^{iHx^0} J^\mu(\vec{x}) e^{-iHx^0} J^\nu(\vec{0}) \right\rangle_{\text{proton}}$$

No Wilson line needed! J^μ is a *physical* current.

$$H = H_0 + \epsilon_x(t) J^\mu(\vec{x}) + \epsilon_0(t) J^\nu(\vec{0})$$

In principle, PDF can be extracted from HT.

Cross Sections

Quantum Computation of Scattering in Scalar Quantum Field Theories

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Cross sections may be determined from the Hadronic tensor:

$$\frac{d^2\sigma}{\mathrm{d}x\,\mathrm{d}y} = \frac{\alpha^2 y}{Q^4} L_{\mu\nu} W^{\mu\nu}$$

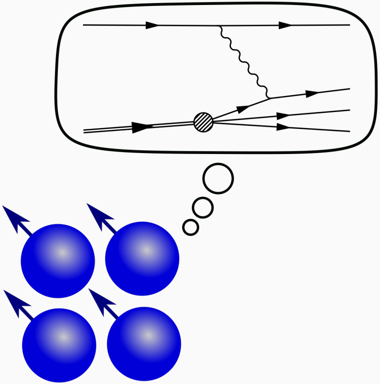
The Future

For QCD PDFs: $\sim 10^6$ qubits needed (20^3 lattice)

Work out exact $S(1080)$ circuits (reliable cost estimates)

Better truncation or improved Hamiltonian could give *small* gains

Understand $1+1$ and $2+1$ bound states?



Outline